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DUPLICATION OF A VERTEX IN EVEN SUM CORDIAL LABELING GRAPHS

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Abstract

An even sum cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,3,\ldots,|V|\}$ such that if an edge uv is assigned the label 1 if $f(u)+f(v)$ is even and the label 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. If a graph has an even sum cordial labeling, it is called an even sum cordial graph. This paper proves that duplication of a vertex of path graph, cycle graph, complete bipartite graph, sub-division graph, bistar graph, shadow graph, square graph, Triangular book graph, wheel graph, crown graph, comb graph, and shell graph are even sum cordial.

1. Introduction

By a graph G = (V, E), we mean a finite, undirected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Harary [6], the origin of graph labeling can be attributed to Rosa [8] and we refer to Gallian [5]. In Cahit [2,3] introduced the concept of cordial labeling of graph. S.Abhirami et al., [4] was introduced Even Sum cordial graph. The brief summaries of definition which are necessary for the present investigation are provided below. In this paper, we prove that duplication of a vertex of path graph, cycle graph, complete bipartitite graph, sub division graph, bistar graph ,shadow graph, square graph, Triangular book graph, wheel graph, crown graph, comb graph and shell graph are even sum cordial .

2. Definitions

Definition 2.1 [4]

Let $G = (V,E)$ be a simple graph and $f: V \to \{1,2,3\ldots.|V|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u) + f(v)$ is even and the label 0 otherwise, f is called an even sum cordial labeling if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with an even sum cordial labeling is called an even sum cordial graph.

Definition 2.2 [7]

Duplication of a vertex V_k of a graph G produces a new graph G1 by adding a vertex $V_k{}'$ with $N(V_k{}') = N(V_k)$.In other words a vertex $V_k{}'$ is said to be a duplication of V_k if all the vertices which are adjacent to V_k are now adjacent to $V_k{}'$.

Definition 2.3 [6]

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A walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices, preceding and following it. No edge appears more than once in a walk. A vertex may appear more than once. Vertices with which a walk begins and ends are called its terminal vertices.

Definition 2.4[6]

A walk is said to be **closed** if the starting vertex and the ending vertex are the same. A walk that is not closed is called an open walk. An open walk in which no vertex appears more than once is known as a path. The number of edges in a path is called the length of a path.

Definition 2.5[6]

A closed walk in which no vertex appears more than once is known as a circuit. A circuit is also called a cycle.

Definition 2.6[6]

A simple graph in which there exists an edge between every pair of vertices is called a complete graph. Since every vertex is joined with every other vertex through an edge, the degree of every vertex is $n-1$ in a complete graph G of n vertices.

Definition 2.7 [6]

A bipartite (bigraph) G is a graph whose point set V can be partitioned into two subsets $V_1$and V_2 such that every line of G joins $V_1$with V_2. If G contains every line joining $V_1$and V_2, then G is a complete bigraph. If V_1 and V_2 have m and n points, we write $G = K_{m,n} = K(m, n)$. A star is a complete bigraph $K_{1,n}$ .Therefore the bigraph $K_{m, n}$ has mn lines. A bigraph is also known as a bipartite graph. The vertex of $K_{1,n}$ with degree n is called the central vertex or apex.

Definition 2. 8 [1]

The subdivision graph $S'(G)$ is obtained from G by subdividing each edge of G with a vertex.

Definition 2.9[1]

The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2 .The graph $P_n \odot K_1$ is called a comb and the graph $C_n \odot K_1$ is called a Crown.

Definition 2.10[1]

A shell graph is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graphs are denoted as $C(n, n-3)$.

Definition 2.11 [1]

One edge union of cycles of same length is called a book graph. The common edge is called base of the book. If we consider t copies of cycles of length $n \geq 3$, then the book is denoted by $B_n^{(t)}$.If $n = 3, 4, 5$ or 6 , then the book B is called book graph with triangular, rectangular, pentagonal or hexagonal pages respectively.

Definition 2.12[10]

The wheel graph W_n is defined to be the joint $k_1 + C_n$. The vertex set corresponding to k_1 is known as apex and vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges.

Definition 2.13[10]

The bistar $B_{n,n}$ is a graph obtained by joining the two copies of $K_{1,n}$ by an edge is called bistar graph.

Definition 2.14 [9]

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking 2 copies of G say G' and G'' . Join each vertex u' in $G'$to the neighbours of the corresponding vertex v' in G'' .

Definition 2.15[9]

For a simple connected graph G, the square of graph G is denoted by G^2 and which is defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Proposition 2.16 [4]

1. Any path P_n is an even sum cordial graph.
2. Any cycle C_n is an even sum cordial graph except $n = 6, 6 + d, 6 + 2d, \ldots$ when $d = 4$.

Proposition 2.17[1]

1. The Subdivision graph $S'(K_{1,n})$ is an even sum cordial graph.
2. The Crown graph $C_n \odot K_1$ is an even sum cordial graph.
3. The Comb graph $P_n \odot K_1$ is an even sum cordial graph.
4. The Triangular book graph is an even sum cordial graph.
5. The Shell graph $C(n, n-3)$ is an even sum cordial graph.
6. The Complete bipartite graph $K_{1,n}$, $K_{2,n}$ and $K_{3,n}$ are even sum cordial graphs.

Proposition 2.18[10]

The bistar graph $B_{n,n}$ is an even sum cordial graph.

In this paper, we denote duplication of a vertex of a Even sum cordial graph G by D(G).

3. Main Results

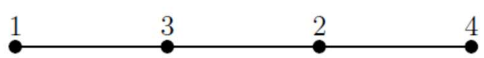
Proposition 3.1

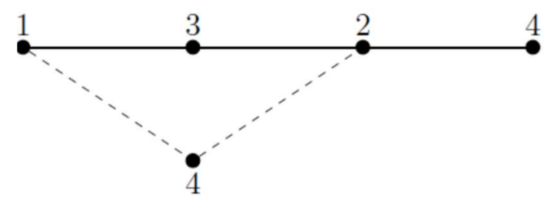
The graph obtained from path P_n by duplicating a vertex is an even sum cordial graph

Proof:

Let D(P_n) be the graph having $n+1$ vertices and $n+1$ edges. Let $v_1, v_2, v_3 \dots v_n$ be the vertices of P_n and v_{n+1} be the duplicating vertex of D (P_n) . First we draw the path P_n by proposition (2.16).We assign the label of v_{n+1} by $n+1$. Next we are constructing a graph D (P_n) by definition (2.2) duplicating any vertex with degree 2 in P_n is an even sum cordial graph.

Example 3.2





Even Sum Cordial Labeling of Path Graph P_4 and its duplication graph

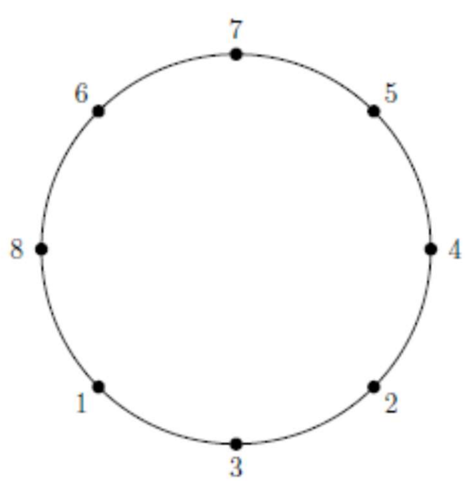
Proposition 3.3

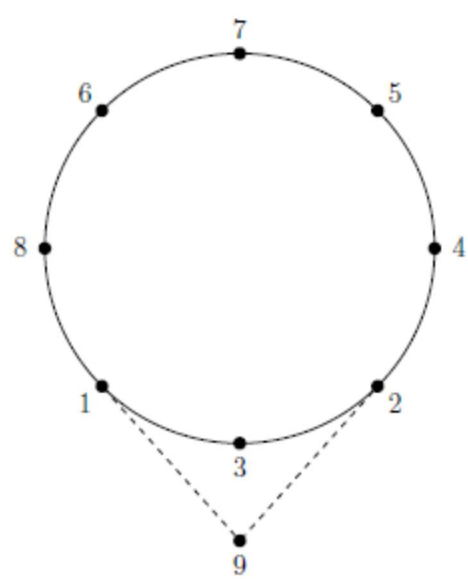
The graph obtained from cycle C_n by duplicating a vertex is an even sum cordial graph when $n > 3$.

Proof:

Let D (C_n) be the graph having $n+1$ vertices and $n+2$ edges. Let $v_1, v_2, v_3 \dots v_n$ be the vertices of C_n and v_{n+1} be the duplicating vertex of D (C_n) . First we draw the cycle C_n by proposition (2.16).We assign the label of v_{n+1} by $n+1$. Next we are constructing a graph D (C_n) by definition (2.2) duplicating any vertex with degree 2 in C_n is an even sum cordial graph.

Example 3.4





Even Sum Cordial Labeling of Cycle graph C_8 and its duplication graph

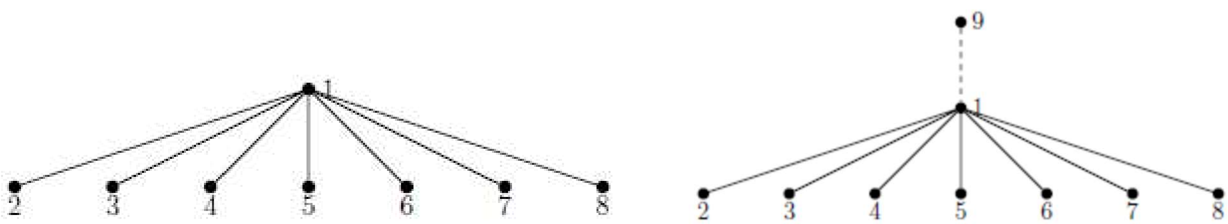
Proposition 3.5

The graph obtained from complete bipartite graph $k_{1,n}$ by duplicating a vertex is an even sum cordial graph

Proof:

Let D $(k_{1,n})$ be the graph having $n+2$ vertices and $n+1$ edges. Let $v_1, v_2, v_3 \dots v_{n+1}$ be the vertices of $k_{1,n}$ and v_{n+2} be the duplicating vertex of D $(k_{1,n})$. First we draw the complete bipartite graph $k_{1,n}$ by proposition (2.17).We assign the label of v_{n+2} by $n+2$. Next we are constructing a graph D $(k_{1,n})$ by definition (2.2) duplicate any vertex with degree 1 in $k_{1,n}$ is an even sum cordial graph.

Example 3.6



Even Sum Cordial Labeling of Complete Bipartite Graph $K_{1,7}$ and its duplication graph

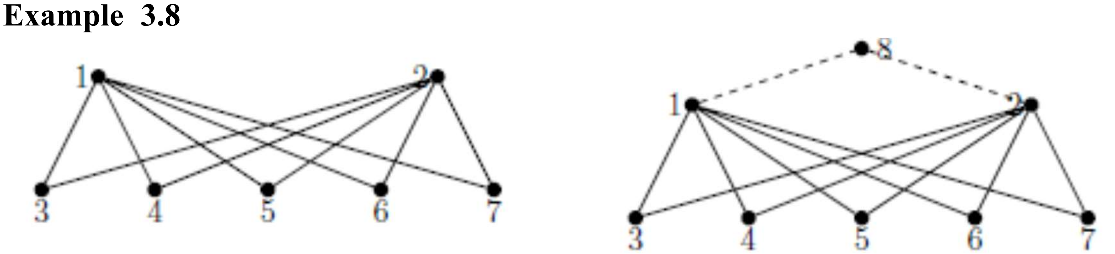
Proposition 3.7

The graph obtained from complete bipartite graph $k_{2,n}$ by duplicating a vertex is an even sum cordial graph

Proof:

Let D$(k_{2,n})$ be the graph having $n+3$ vertices and $2n+2$ edges. Let $v_1, v_2, v_3 \dots v_{n+2}$ be the vertices of $k_{2,n}$ and v_{n+3} be the duplicating vertex of D $(k_{2,n})$. First we draw the complete bipartite graph $k_{2,n}$ by proposition (2.17).We assigns the label of v_{n+3} by $n+3$. Next we are constructing a graph D $(k_{2,n})$ by definition (2.2) duplicate any vertex with degree 2 in $k_{2,n}$ is an even sum cordial graph.

Example 3.8



Even Sum Cordial Labeling of Complete Bipartite Graph $k_{2,5}$ and its duplication graph

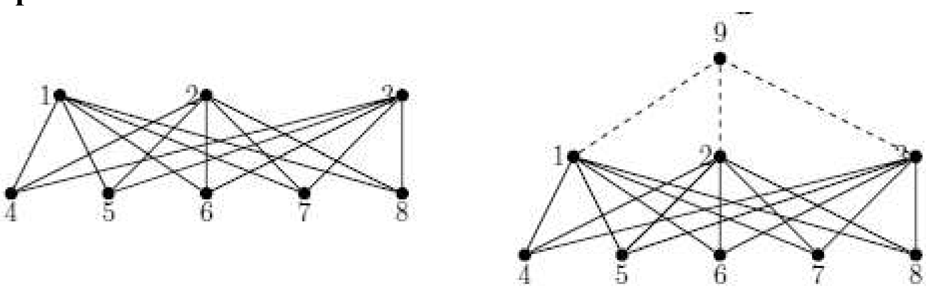
Proposition 3.9

The graph obtained from complete bipartite graph $k_{3,n}$ by duplicating a vertex is an even sum cordial graph

Proof:

Let D $(k_{3,n})$ be the graph having $n+4$ vertices and $3n+3$ edges. Let $v_1, v_2, v_3 \ldots v_{n+3}$ be the vertices of $k_{3,n}$ and v_{n+4} be the duplicating vertex of D $(k_{3,n})$. First we draw the complete bipartite graph $k_{3,n}$ by proposition (2.17).We assigns the label of v_{n+4} by $n+4$. Next we are constructing a graph D $(k_{3,n})$ by definition (2.2) duplicating any vertex with degree 3 in $k_{3,n}$ is an even sum cordial graph

Example 3.10



Even Sum Cordial Labeling of Complete Bipartite Graph $k_{3,5}$ and its duplication graph

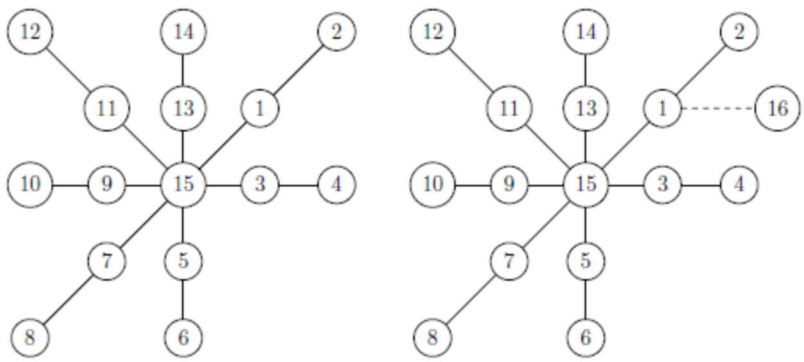
Proposition 3.11

The graph obtained from subdivision graph $S'(K_{1,n})$ by duplicating a vertex is an even sum cordial graph

Proof:

Let D($S'(K_{1,n})$) be the graph having $2n+2$ vertices and $2n+2$ (or) $2n+1$ edges. Let $v_1, v_2, v_3 \ldots v_{2n+1}$ be the vertices of $S'(K_{1,n})$ and v_{2n+2} be the duplicating vertex of D($S'(K_{1,n})$. First we draw the subdivision graph $S'(K_{1,n})$ by proposition (2.17). We assign the label of v_{2n+2} by $2n+2$. Next we are constructing a graph D ($S'(K_{1,n})$) by definition (2.2) duplicate any vertex with degree 1(or) degree 2 in $S'(K_{1,n})$ is an even sum cordial graph.

Example 3.12



Even Sum Cordial Labeling of Subdivision Graph of $(\boldsymbol{K_{1,7}})$ and its duplication graph

Proposition 3.13

The graph obtained from bistar graph $B_{n,n}$ by duplicating a vertex is an even sum cordial graph when n is an odd integer.

Proof:

Let D $(B_{n,n})$ be the graph having $2n+3$ vertices and $2n+2$ edges. Let $v_1, v_2, v_3 \dots v_{2n+2}$ be the vertices of $B_{n,n}$ and v_{2n+3} be the duplicating vertex of D $(B_{n,n})$. First we draw the bistar graph $B_{n,n}$ by proposition (2.18).We assigns the label of v_{2n+3} by $2n+3$. Next we are constructing a graph D $(B_{n,n})$ by definition (2.2) duplicating any vertex with degree 1 in $B_{n,n}$ is an even sum cordial graph.

Example 3.14

Even Sum Cordial Labeling of Bistar Graph $\boldsymbol{B_{3,3}}$ and its duplication graph

Proposition 3.15

The shadow graph $D_2(B_{n,n})$is an even sum cordial graph.

Proof:

Let G be the shadow graph $D_2(B_{n,n})$ having $4(n+1)$ vertices and $4(2n+1)$ edges. Consider two copies of $B_{n,n}$ we take one is $B_{n,n}$ and another one is $B_{n,n}{}'$.

Let $x, y, x_1, x_2, x_3, \dots, x_n, y_1, y_2, y_3, \dots, y_n, x', y', x_1', x_2', x_3', \dots, x_n', y_1', y_2', y_3', \dots, y_n'$ be vertices of G. Consider $\{x, y, x_i, y_i \ , 1 \le i \le n\}$ and$\{x' \ , y', x_i', y_i' \ , 1 \le i \le n\}$are vertex sets of $B_{n,n}$ and $B_{n,n}{}'$ respectively.

Now we assign the labeling of vertices as follows.

Define f: $V(G) \to \{1,2,3, \dots 4(n+1)\}$

Let p_1 be the highest odd number and $p_2$be the highest even number.

$f(x) = 2;$ $\qquad f(x') = 1;$

$f(y) = p_2;$ $\qquad f(y') = p_1;$

$f(x_i) = 2(i+1) \ ; \quad 1 \le i \le n$

$f(x_{i-2}') = 2i+2 \ ; \quad n+1 \le i \le n+2$

$f(y_i) = 2i+1 \ ; \quad 1 \le i < n+1$

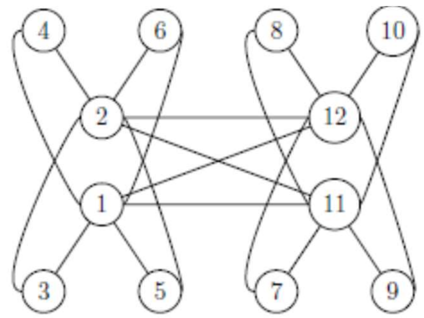
$f(y_{i-2}') = 2i+1 \ ; \quad n+1 \le i \le n+2$

First we construct the graph $B_{n,n}$ and $B_{n,n}{}'$ by Definition 2.13. Then we draw the shadow graph $D_2(B_{n,n})$ by Definition 2.14 join each vertex in $B_{n,n}$ to the neighbours of the corresponding vertex in $B_{n,n}{}'$ .Thus we obtained the graph G.

In view of the above defined labeling pattern, We have $e_f(0) = e_f(1) = 4n+2$.

$\therefore \left|e_f(0) - e_f(1)\right| \leq 1$ .Hence G is an even sum cordial graph.

Example 3.16



Even Sum Cordial Labeling of Shadow Graph $D_2(B_{2,2})$

Proposition 3.17

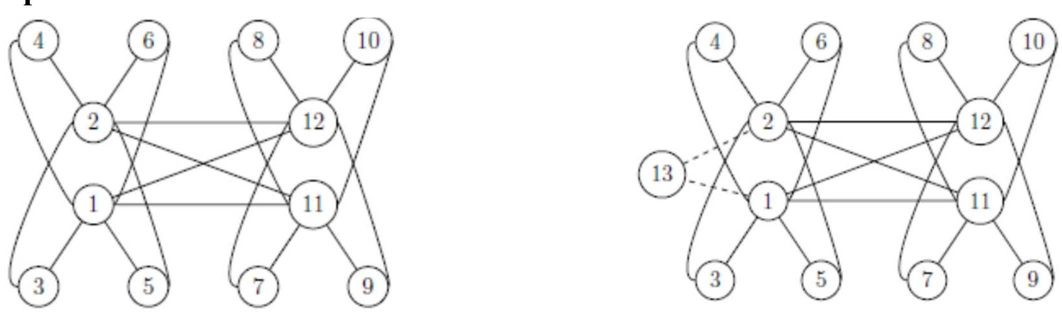
The graph obtained from shadow graph $D_2(B_{n,n})$ by duplicating a vertex is an even sum cordial graph.

Proof:

Let D $(D_2(B_{n,n}))$ be the graph having $4n+5$ vertices and $8n+6$ edges. Let $v_1, v_2, v_3 \dots v_{4n+4}$ be the vertices of the graph $D_2(B_{n,n})$ and v_{4n+5} be the duplicating vertex of D $(D_2(B_{n,n}))$.

First we draw the shadow graph $D_2(B_{n,n})$by proposition (3.15).We assigns the label of v_{4n+5} by $4n+5$. Next we are constructing a graph D $(D_2(B_{n,n}))$by definition (2.2) duplicating any vertex with degree 2 in $D_2(B_{n,n})$ is an even sum cordial graph.

Example 3.18



Even Sum Cordial Labeling of Shadow Graph $\boldsymbol{D_2(B_{2,2})}$ and its duplication graph

Proposition 3.19

The square graph $B_{n,n}{}^2$ is an even sum cordial graph.

Proof:

Let G be the square graph $B_{n,n}{}^2$ having $2n+2$ vertices and $4n+1$ edges.

Let $x, y, x_1, x_2, x_3, \dots x_n,\ y_1, y_2, y_3, \dots y_n$ be the vertices of G.

Now assign the labeling of vertices as follows.

$f\colon V(G) \rightarrow \{1,2,3,\dots 2n+2\}$

$f(x) = 1$; $f(y) = 2$;

$f(x_i) = 2i + 1$; $1 \leq i \leq n$
$f(y_i) = 2(i + 1)$; $1 \leq i \leq n$

First we draw the bistar graph $B_{n,n}$ by Definition 2.13.Then we draw the square graph $B_{n,n}{}^2$ by Definition 2.15 join any two vertices by an edge if and only if there is a path of length at most 2 between them.

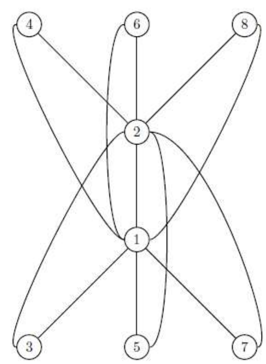
We construct the edges as follows.

$E(G) = \{xy, xx_i, yy_i, xy_i, yx_i, 1 \leq i \leq n\}$

In the view of above labeling pattern, we have $e_f(0) = 2n + 1$; $e_f(1) = 2n$.

$\therefore \left|e_f(0) - e_f(1)\right| \leq 1$. Hence $B_{n,n}{}^2$ is an even sum cordial graph.

Example 3.20



Even Sum Cordial Labeling of Square Graph $B_{3,3}{}^2$

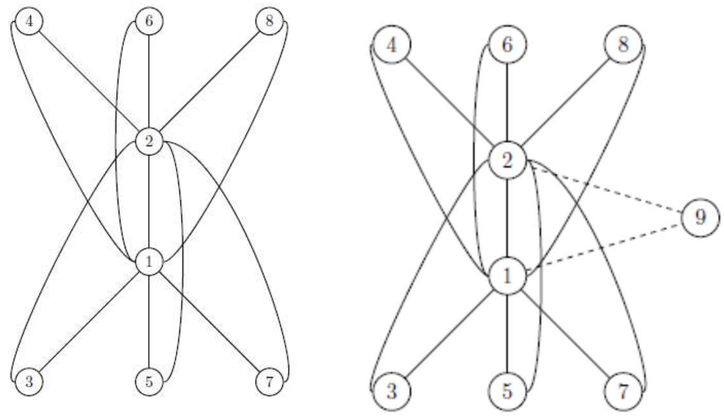
Proposition 3.21

The graph obtained from square graph $B_{n,n}{}^2$ by duplicating a vertex is an even sum cordial graph.

Proof:

Let D ($B_{n,n}{}^2$) be the graph having $2n + 3$ vertices and $4n + 3$ edges. Let $v_1, v_2, v_3 \dots v_n$ be the vertices of $B_{n,n}{}^2$ and v_{2n+3} be the duplicating vertex of D ($B_{n,n}{}^2$). First we draw the square graph $B_{n,n}{}^2$ by proposition (3.19).We assigns the label of v_{2n+3} by $2n + 3$. Next we are constructing a graph D ($B_{n,n}{}^2$) by definition (2.2) duplicating any vertex with degree 2 in $B_{n,n}{}^2$ is an even sum cordial graph.

Example 3.22



Even Sum Cordial Labeling of Square Graph $B_{3,3}{}^2$ and its duplication graph

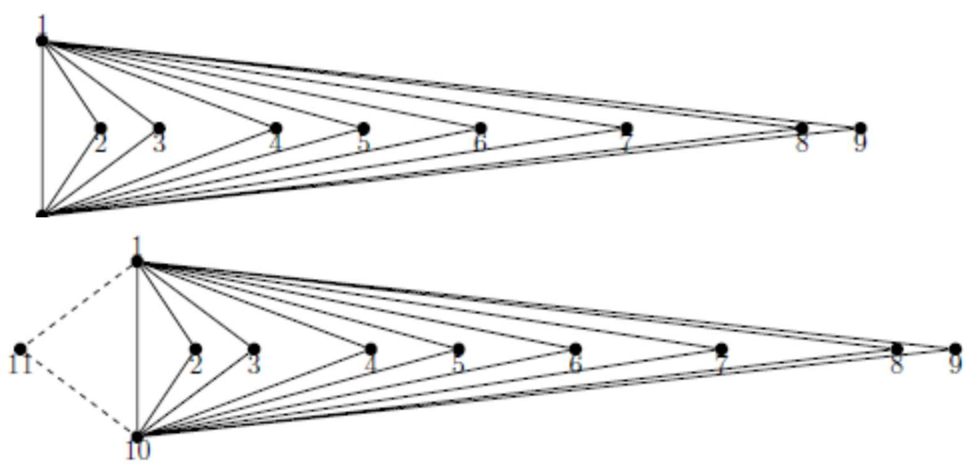
Proposition 3.23

The graph obtained from a triangular book graph by duplicating a vertex is an even sum cordial graph when n is an even integer.

Proof:

Let D (B) be the graph having $n+1$ vertices and $2n-1$ edges. Let $v_1, v_2, v_3 \dots v_n$ be the vertices of book graph B and v_{n+1} be the duplicating vertex of D (B). First we draw the triangular book graph with n vertices by proposition (2.17).We assigns the label of v_{n+1} by $n+1$. Next we are constructing a graph D (B) by definition (2.2) duplicating any vertex with degree 2 in triangular book graph is an even sum cordial graph

Example 3.24



Even Sum Cordial Labeling of Triangular Book Graph with 10 vertices and its duplication graph

Proposition 3.25

The wheel graph W_n is an even sum cordial graph, if n is an odd integer.

Proof:

Let G be the wheel graph W_n.

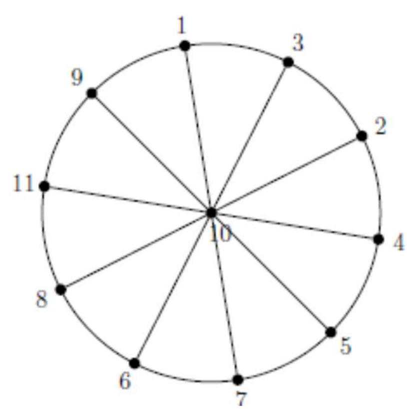
Let v_0 be the apex vertex of wheel W_n and $v_1, v_2, v_3 \dots v_n$ be the rim vertices.
Define $f: V(G) \rightarrow \{1,2,3 \dots n\}$.
We consider the following two cases for labeling pattern.
Case 1:
If $n \equiv 1 \ (mod\ 4)$, we assign the label values in the following ways
Define for $i = 1,5,9 \dots n-4$.

$f(v_i) = i$;
$f(v_{i+1}) = i+2$;
$f(v_{i+2}) = i+1$;
$f(v_{i+3}) = i+3$ and
$f(v_0) = n$;

Case 2:
If $n \equiv 3 \ (mod\ 4)$,We assign the label values in the following ways
Define for $i = 1,5,9 \dots n-6$.

$f(v_i) = i$;
$f(v_{i+1}) = i+2$;
$f(v_{i+2}) = i+1$;
$f(v_{i+3}) = i+3$ and
$\{ f(v_{n-2}) = n, f(v_{n-1}) = n-2, f(v_0) = n-1 \}$;

First we draw a circle by using the above labeling pattern then join the apex vertex v_0 to all rim vertices by an edge. Thus, we get $e_f(0) = n-1 = e_f(1)$.
$\therefore |e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph.
Example 3.26

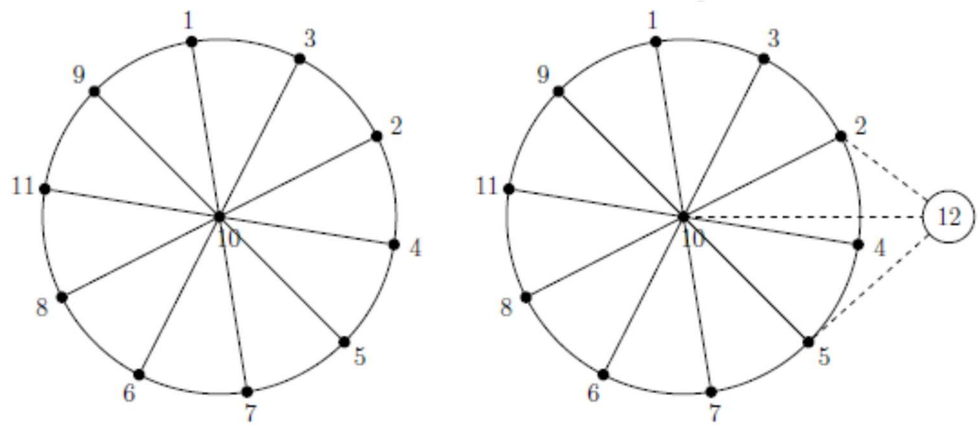


Even Sum Cordial Labeling of Wheel Graph W_{11}

Proposition 3.27
The graph obtained from wheel graph W_n by duplicating a vertex is an even sum cordial graph when n is an odd integer.
Proof:

Let D(W_n) be the graph having $n+1$ vertices and $2n+1$ edges. Let $v_1, v_2, v_3 \ldots v_n$ be the vertices of W_n and v_{n+1} be the duplicating vertex of D(W_n). First we draw the graph W_n with n vertices by proposition (3.25).We assign the label of v_{n+1} by $n+1$. Next we are constructing a graph D (W_n) by definition (2.2) duplicating any vertex with degree 3 in W_n is an even sum cordial graph.

Example 3.28



Even Sum Cordial Labeling of Wheel Graph W_{11} and its duplication graph

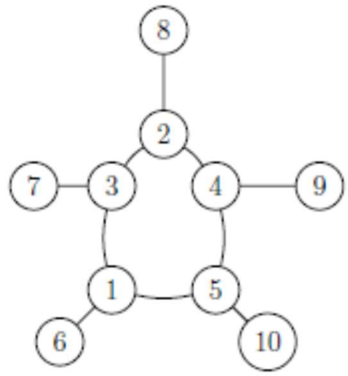
Proposition 3.29

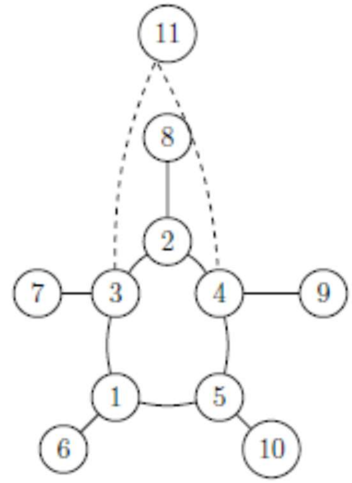
The graph obtained from crown graph $C_n \odot K_1$ by duplicating a vertex is an even sum cordial graph when n is an odd integer.

Proof:

Let D($C_n \odot K_1$) be the graph having $2n+1$ vertices and $2n+3$ edges. Let $v_1, v_2, v_3 \ldots v_{2n}$ be the vertices of the graph $C_n \odot K_1$and v_{2n+1}be the duplicating vertex of D ($C_n \odot K_1$). First we draw the graph $C_n \odot K_1$with 2n vertices by proposition (2.17).We assign the label of v_{2n+1} by $2n+1$. Next we are constructing a graph D($C_n \odot K_1$) by definition (2.2) duplicating any vertex with degree 3 in $C_n \odot K_1$ is an even sum cordial graph.

Example 3.30





Even Sum Cordial Labeling of Crown Graph $\boldsymbol{C_5} \odot \boldsymbol{K_1}$ and its duplication graph

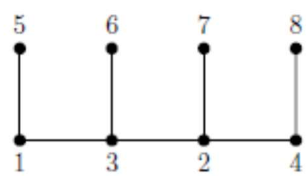
Proposition 3.31

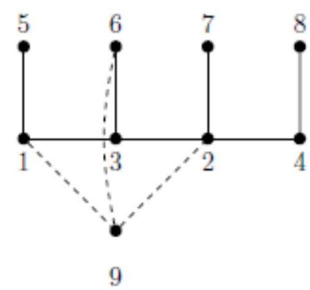
The graph obtained from comb graph $P_n \odot K_1$ by duplicating a vertex is an even sum cordial graph.

Proof:

Let D($P_n \odot K_1$) be the graph having $2n+1$ vertices and $2n+2$ edges. Let $v_1, v_2, v_3 \ldots v_{2n}$ be the vertices of the graph $P_n \odot K_1$ and v_{2n+1} be the duplicating vertex of D($P_n \odot K_1$). First we draw the graph $P_n \odot K_1$ with 2n vertices by proposition (2.17). We assign the label of v_{2n+1} by $2n+1$.Next we are constructing a graph D($P_n \odot K_1$) by definition (2.2) duplicating any vertex with degree 3 in $P_n \odot K_1$ is an even sum cordial graph.

Example 3.32





Even Sum Cordial Labeling of Comb Graph $\boldsymbol{P_4} \odot \boldsymbol{K_1}$ and its duplication graph

Proposition 3.33

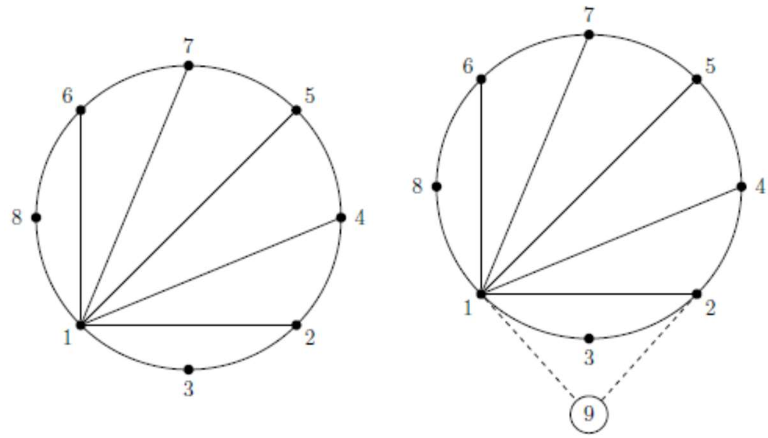
The graph obtained from Shell graph $C(n, n-3)$ by duplicating a vertex is an even sum cordial graph when n is an even integer.

Proof:

Let D($C(n, n-3)$) be the graph having $n+1$ vertices and $2n-1$ edges. Let $v_1, v_2, v_3 \ldots v_n$ be the vertices of $C(n, n-3)$and v_{n+1} be the duplicating vertex of D($C(n, n-3)$).

First we draw the graph $C(n, n-3)$with n vertices by proposition (2.17). We assign the label of v_{n+1} by $n+1$. Next we are constructing a graph D(C $(n, n-3)$) by definition (2.2) duplicating any vertex with degree 2 in $C(n, n-3)$ is an even sum cordial graph.

Example 3.34



Even Sum Cordial Labeling of Shell Graph $\boldsymbol{C}(8,5)$ and its duplication graph

4. Conclusion

In this paper, we proved that duplication of a vertex of path graph, cycle graph, complete bipartite graph, sub division graph,bistar graph, shadow graph, square graph, Triangular book graph, wheel graph, crown graph, comb graph, and shell graph are even sum cordial.

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